Standard-Essential Patents and Incentives for Innovation

Chayanin Wipusanawan*

August 2023

Abstract

The paper presents a model of innovation with multiple independently developed technologies, whose individual values depend on other technologies. The main application is technology standards involving many patented inventions. I characterize the value-sharing rule that maximizes the expected social value of innovation and compare the associated second-best optimal investments to the equilibrium investments under a competitive benchmark. In the presence of both potential complements and substitutes among technologies, the competitive benchmark fails to achieve efficiency. My results imply that the rule recently endorsed by courts and policymakers to determine royalties for standard-essential patents provides suboptimal innovation incentives.

JEL classification: L15, O31, O34, O38

Keywords: innovation, standardization, standard-essential patents, FRAND, complements

1 Introduction

Many modern products combine multiple inventions that are protected by patents. Technical standards in the information and communication technology industry are a

* University of Passau, School of Business, Economics and Information Systems. E-mail: chayanin.wipusanawan@uni-passau.de. I am grateful to my doctoral supervisors Florian Schuett and Bert Willems for their tremendous guidance and support. I also thank Cédric Argenton, Jan Boone, Eric van Damme, Peter Kort, Gerard Llobet, Jens Prüfer, and Katharine Rockett, as well as conference and seminar participants at Tilburg University, Toulouse Applied Theory Workshop, EARIE 2021, and TILEC Workshop on the Economics of Patents and Standards for their helpful comments and suggestions.

prime example. The LTE standard for mobile communications, for instance, has over 10,000 patents that are claimed as essential to the implementation of the standard (Baron and Pohlmann, 2018). If a patented invention is necessary to implement the standard, the patent becomes a *standard-essential patent* (SEP) and the implementer needs to obtain a license to market its products.

When many technological inputs are available and multiple are combined to make a product, technologies from different innovators may be competing with each other, with the extreme case of them being perfect substitutes. Alternatively, they may complement each other meaning that the whole is greater than the sum of its parts, with the extreme case being perfect complements. In this paper, I study how the individual innovators should be rewarded when there are multiple technologies whose values depend on which other technologies are available, under the assumption that only the innovation outcome is observable but not research investment. I also investigate how well the outcome arising from a competition between the technologies compares to the optimal incentives. The optimal incentive scheme induces research investments that maximize the expected value of the technologies net of the research costs, restricted by a budget constraint that the revenues given to innovators come from the values that they jointly create. In other words, external subsidies are not allowed. The incentive scheme is therefore a sharing rule for the value created by the technologies.

If each innovator is paid the incremental value of the technology (i.e. the difference between the values with and without the technology) then the innovation efforts would be socially optimal. This follows the conventional economic wisdom that decisions are efficient if the marginal private incentive aligns with the marginal social contribution.[1] However, paying every innovator the incremental value is not always feasible given the budget constraint. With complementary technologies, the sum of individual incremental values could exceed the total value of the technologies, as discussed by, for example, Shapiro (2007). With the binding budget constraint, the "second-best" incentives mirror the Ramsey pricing solution from the budget-balanced multi-product pricing problem (Baumol and Bradford, 1970).

The competitive royalty that a patent holder can command if the patent holders

[1] Pigouvian taxes (or subsidies) for externalities are one of the best-known applications of this wisdom. In the work that introduces this concept, Pigou (1920, 161) uses patent laws as an example of a tool for "bringing marginal trade net product and marginal social net product more closely together".

engage in a price competition after the technologies are invented is restricted by the incremental value of the technology. However, the idea also applies to any arbitrary group of technologies. Their combined competitive prices are restricted by their joint incremental value over the best alternative that does not contain any of their technologies. This means when there are complements, the competitive royalties will be strictly smaller than the individual incremental values. When there are possibilities of both complements and substitutes, then the incentives given by competitive royalties are not second-best, and allowing royalties greater than the competitive level can improve the expected social welfare. By imposing competitive royalties within a specific realized state of innovation, it is possible that an inventor has a competing substitute that drives down its competitive royalty (justifiably in isolation) in one state of the world, but receives a competitive royalty smaller than its incremental value in another state due to complementarities. By allowing supra-competitive royalties in the former state, the marginal incentive in expectation over different states moves closer to the marginal social contribution when the incentives are considered in expectation over multiple possible outcomes.

In the context of SEPs, the outcome from the competition between patent holders has been proposed as the benchmark for patent royalties that the SEP holders can charge under the so-called *fair, reasonable, and non-discriminatory* (FRAND) requirement that is often imposed by standard-setting organizations (SSOs).[2] Determining whether a licensing term is FRAND is a contentious issue in patent litigation and competition law enforcement.[3] An influential interpretation of the "fair and reasonable" part of FRAND is that royalties should reflect the outcome of a hypothetical competition before the standard is set (Swanson and Baumol, 2005),[4] which is said to equal the *incremental value* of the patented invention (Farrell et al., 2007). This interpretation has been endorsed in the United States by the Federal Trade

[2] Lemley (2002) studies the policies of forty-three SSOs in telecommunications and computer networking industries and finds that the majority of them require FRAND licensing.

[3] For example, in the case *Microsoft Corp. v. Motorola, Inc.*, 696 F.3d 872 (9th Cir. 2012), the US court ruled that the royalties that Motorola demanded for every Wi-Fi-enabled device were excessive and in breach of Motorola's FRAND licensing commitment. The royalties set by the court were substantially lower than the initial offer. See also Pentheroudakis and Baron (2017) and Siebrasse and Cotter (2017) for summaries of cases from multiple jurisdictions in which the court determined the FRAND royalty rate.

[4] This is often called *ex ante* competition in the literature, since it reflects the situation before the standard is chosen. I do not use this terminology in the paper to avoid confusion, since such competition occurs before standardization but *after* the innovation process, which is the focus of this paper.

Commission (2011) and courts.[5]

Other than the hypothetical competition or incremental value, a few other interpretations of FRAND have been proposed. One of the simplest sharing rules proposed is *numeric proportionality*, where each patent earns its owner an equal share of the total royalty. Numeric proportionality has been adopted by some patent pools and put forth as an interpretation of FRAND royalties (Layne-Farrar et al., 2007; Layne-Farrar and Lerner, 2011). Another approach is to adopt the Shapley value (Shapley, 1953) or a similar axiomatic approach to define FRAND (Layne-Farrar et al., 2007; Dehez and Poukens, 2014; Hougaard et al., 2023). While this paper does not formally evaluate these approaches, they do not generally coincide with the second-best scheme characterized in this paper.

This paper contributes to the economic literature of how innovation should be incentivized in the standardization context. The model adds the innovation stage to models of the standard-setting process that begin after the technologies have been invented. Lerner and Tirole (2015) formally model standard selection and the notion of the pre-standardization competition, and the competitive benchmark in this paper follows the definition of competitive equilibrium in their paper. A similar model of innovation incentives is used by Layne-Farrar et al. (2014) to show that competitive royalties are not sufficient to attract innovating firms to participate in standardization efforts when the choice of participation is endogenous. Other papers study different aspects of innovation incentives under FRAND licensing commitments. Layne-Farrar and Llobet (2014) show that if the values are multidimensional and heterogeneous, there is not necessarily a single incremental value that all parties can agree upon and that the hypothetical competition may not select the optimal standard. Ganglmair et al. (2012) study the enforcement of FRAND commitments through competition law and argue that damage remedies against SEP holders suboptimally restrict innovation. Dewatripont and Legros (2013) show that FRAND licensing requirements may lead to firms claiming SEPs that are not really "essential" to the standard.

More broadly, this paper is related to the literature on rewarding complex technolo-

[5] For example, in *Ericsson, Inc. v. D-Link Systems, Inc.*, 773 F.3d 1201 (Fed. Cir. 2014), the court notes that "the patentee's royalty must be premised on the value of the patented feature, not any value added by the standard's adoption of the patented technology ... to ensure that the royalty award is based on the incremental value that the patented invention adds to the product, not any value added by the standardization of that technology".

gies, especially complementary technologies. I focus on the scheme that distributes the fixed social value of the technologies and abstracts away from licensing and downstream competition.[6] The set-up of a static innovation game under a pre-determined incentive scheme is similar to the literature on research contests and procurement, e.g. Taylor (1995) and Che and Gale (2003), but these papers focus on competing (substitute) firms and do not include complements. For complementary technologies, Gilbert and Katz (2011) study how to divide the value of multiple perfectly complementary technologies among inventors. Their paper uses a dynamic model in which inventors compete to discover each component sequentially. In contrast to their work, my model does not feature a "winner-takes-all" race to discover a technology, but allows each invention to potentially have complementary as well as substitute innovations.

The structure of innovation efforts in this paper can also be compared to the standard problem of incentives for teams, in which multiple agents contribute to a common goal. The model of multiple innovators contributing to the expected social welfare corresponds to the team incentive model of Holmström (1982), with the key difference being that, instead of contracting solely on the joint outcome, the principal can also contract upon individual signals (i.e. research outcomes), even though actions of individual agents remain non-contractible.

The rest of the paper is structured as follows. To set the stage, Section 2 presents a simplified example that highlights the intuition presented in this paper. Section 3 describes the model set-up for the rest of the paper, while Section 4 explains the result that the competitive benchmark is not necessarily optimal. Section 5 concludes with discussions on the policy implications and possible caveats of the model's findings.

2 Example

Consider a standard that requires two perfectly complementary parts A and B and has a value of 1 for the consumer. For each part A or B, there are two firms that independently try to invent a technology for the part. Let $\{A_1, A_2\}$ be the set of firms working on part A and $\{B_1, B_2\}$ the set of firms working on part B. The two firms who work on the same part pursue different but equally good technologies. The

[6] See e.g. Ménière (2008), Denicolò and Halmenschlager (2012), Biagi and Denicolò (2014), D'Antoni and Rossi (2014), Choi and Gerlach (2015), and Denicolò and Zanchettin (2022) for the licensing of complementary technologies.

innovation process is probabilistic: the probability that firm i successfully invents its technology depends on its costly innovation efforts. There are therefore sixteen possible states of the world depending on whether each of the four firms' technology is invented. A standard can be created if at least one technology is invented for each part. Otherwise, the technologies have zero value on their own. The value of the technologies in each state of the world can be described as a function of the set S of successful technologies

$$v(S) = \begin{cases} 1 & \text{if } S \cap \{A_1, A_2\} \neq \varnothing \text{ and } S \cap \{B_1, B_2\} \neq \varnothing \\ 0 & \text{otherwise.} \end{cases}$$

We are interested in setting the incentives for the firms to maximize the expected social welfare, defined as the expected value of the technologies net of total cost of efforts. The firms' efforts are not contractible, but an incentive scheme can condition the payment to each firm on the state of the world. In addition, assume that there is no possibility of subsidies, so the incentives must be provided by distributing the values created by the technologies. Firms choose their efforts to maximize their expected profit given the incentive scheme.

For this section, the effort costs of the firms are not modeled explicitly. However, we will take as given that firms choose the unconstrained welfare-maximizing efforts if each of them is paid the incremental value of its technology in each state, where the incremental value of a technology is defined as the difference between the value of the standard in that state and the value in the state that the technology does not exist: $v(S) - v(S \setminus \{i\})$. This follows the economic intuition that an economic agent behaves optimally if they fully internalize their marginal social contributions.

Given the assumption of no subsidies, it is not necessarily possible to pay each firm its incremental value in each state. The incremental value of a technology in a particular state is either 1 or 0, depending on whether the technology is pivotal for the standard or not. The middle column of Table 1 summarizes the incremental values in different types of states of the world in which the standard is created. For example, if all firms but A_2 successfully invent their technologies, then A_1's technology is pivotal since the standard cannot be created without it. Its incremental value is therefore the whole value 1, whereas the incremental value for each of B_1 and B_2's technologies is 0. If instead exactly one technology is invented for each part, for example A_1 and B_1's, then both invented technologies are pivotal, meaning

Table 1: The incremental values and the competitive prices in different states of the world

Invented technologies				Incremental values					Competitive prices				
				A_1	A_2	B_1	B_2	Total	A_1	A_2	B_1	B_2	Total
A_1	A_2	B_1	B_2	0	0	0	0	0	0	0	0	0	0
A_1		B_1	B_2	1		0	0	1	1		0	0	1
A_1		B_1		1		1		2	½		½		1

that the incremental value of each technology is the full value of 1. In this case, it would not be possible for our incentive mechanism to pay each firm its incremental value with the value of the technologies in that state.

One mechanism that we consider is one whereby the payments the firms receive are determined by price competition after the state of the world is realized, which is the economic interpretation of "reasonable" royalties as proposed by Swanson and Baumol (2005) and formalized by Lerner and Tirole (2015). In this competitive process, each successful firm simultaneously announces a price for its technology to the user. The user then observes the prices and chooses the set of technologies to buy. The competitive prices are shown in the right column of Table 1. Through the usual Bertrand argument, if two firms working on the same part, A or B, are both successful, perfect substitutability will drive the equilibrium price down to zero. If there is one pivotal firm in the realized state of the world, then with the take-it-or-leave-it offer the firm that owns the pivotal technology can command the whole value of the standard. In this case, the competitive prices coincide with the incremental values of the technologies. However, when there are two pivotal technologies, for example when only A_1 and B_1 are invented, the total payment that these two firms can jointly command is the value of the standard, since the user will not be willing to pay in total more than the value. In this case, the symmetric competitive price is ½ for each technology.

Can a social planner who wishes to maximize the expected social welfare do better than the competitive prices given the budget constraints? Although firms are paid less than the incremental values of their technologies in the states of the world in which two technologies are pivotal, it is not possible to pay more in those states without breaking the budget constraints. However, the planner can bring the *expected*

Table 2: The competitive prices in different states of the world in cases of perfect substitutes and perfect complements

Invented technologies	Substitutes				Complements			
	C	D	Total	Value	C	D	Total	Value
Ⓒ Ⓓ	0	0	0	1	½	½	1	1
Ⓒ	1		1	1	0		0	0

incentives closer to the ideal case of paying incremental values. In the state of the world in which all firms successfully invent their technologies, the competitive price for each technology is zero, so the budget is not used up in this state. By increasing the payment for each firm in this state by $\varepsilon > 0$, the planner brings the expected payment that each firm receives when its innovation succeeds closer to the ideal level that is given by the incremental value. Paying more than the incremental value in the state where it is possible compensates for the insufficient payment in other states.

We can contrast this example to simpler cases of two technologies that are either only perfect complements or perfect substitutes, as shown in Table 2. Suppose there are two firms C and D, similarly working on their own technologies. If the two technologies are perfect substitutes, such that only one is needed for the standard, then the competitive price is the full value 1 if only one firm successfully invents and 0 if both firms succeed. This is perfectly in line with the incremental values, so the competitive prices lead to the socially optimal efforts. On the other hand, if C and D are perfect complements, such that the standard has value only if both firms succeed, then we have again the problem of two pivotal technologies and the competitive price for each technology is ½, which is smaller than the incremental value. However, in this case the social planner cannot increase the incentives, since there is no longer leftover budget in any states of the world.

3 The model

Consider the following model of innovation in which multiple technologies can be invented. There is a set $N = \{1, \ldots, n\}$ of risk-neutral firms. Each firm is endowed with an idea for a single technology. Firm i successfully invents its technology with probability x_i if it invests $c_i(x_i)$. Assume that each function c_i is increasing and convex with $c_i(0) = 0$ and $c_i'(0) = 0$. Let x denote the vector of *research efforts* $(x_1, \ldots, x_n)$. All firms choose their research efforts simultaneously.

Given the binary outcome of each technology, there are 2^n possible states of the world, each state characterized by the set of successfully invented technologies. I will refer to the state of the world in which S is the set of successfully invented technologies as state S. The set of all possible states is the power set of N, denoted $\mathcal{P}(N)$.

If S is the set of available technologies, the technologies jointly create a value of $v(S)$ to the user. The value function v is assumed to be monotonic, meaning for any sets S and T, we have $v(T) \leq v(S)$ if $T \subseteq S$, and normalized such that $v(\varnothing) = 0$.[7]

Let $\bar{v}(x)$ denote the expected value of $v(S)$ given the research efforts x,

$$\bar{v}(x) = \sum_{S \in \mathcal{P}(N)} \Pr(S \mid x)\, v(S),$$

where

$$\Pr(S \mid x) = \prod_{i \in S} x_i \prod_{j \in N \setminus S} (1 - x_j)$$

is the probability that state S is realized given the research efforts x. The social objective function is the expected net value of innovation

$$w(x) = \bar{v}(x) - \sum_{i \in N} c_i(x_i).$$

As the *first-best* benchmark, let x^* be the research efforts that maximize the expected social surplus $w(x)$. Assume that x^* is the unique local and global maximum that is an interior point of $[0, 1]^n$. The first-best research efforts x^* must be a solution to the first-order conditions

$$\frac{\partial \bar{v}(x^*)}{\partial x_i} = c_i'(x_i^*) \tag{1}$$

for all $i \in N$. The marginal contribution of the effort is the expected incremental value

$$\frac{\partial \bar{v}(x)}{\partial x_i} = \sum_{S \in \mathcal{P}(N)} \Pr(S \setminus \{i\} \mid x_{-i}) \left[v(S) - v(S \setminus \{i\})\right]$$

where $x_{-i} = (x_1, \ldots, x_{i-1}, 0, x_{i+1}, \ldots, x_n)$. For S such that $i \in S$, the probability

[7] It is conceivable that including more technologies into a standard may reduce the value of the standard, meaning that the value function of the standard is not monotonic (see Lerner and Tirole, 2015). Since the standard selection process is not modeled in this paper, the monotonicity of v can be justified by interpreting $v(S)$ as the value of the best standard that can be chosen in state S. In other words, invented technologies can be ex post freely disposed of.

$\Pr(S \setminus \{i\} \mid x_{-i})$ can be interpreted as the probability that other firms in set S—and only them—succeed independently of firm i's effort, and we have

$$\Pr(S \mid x) = x_i \Pr(S \setminus \{i\} \mid x_{-i}). \tag{2}$$

Throughout this paper, the research efforts x are assumed to be unobservable, but it is possible to specify how much each firm is paid in a particular state of the world. Let $r_i(S) \geq 0$ denote the revenue that firm i receives in state S. A full schedule of revenues for all i and S is referred to as a *revenue scheme*. I will also use $r(S)$ and r to denote the revenues for firms in state S and the entire revenue scheme. Given a revenue scheme, the innovation decisions can then be characterized as a strategic game with n firms, in which each firm i chooses its research effort x_i to maximize its expected profit

$$\sum_{S \in \mathcal{P}(N)} \Pr(S \mid x)\ r_i(S) - c_i(x_i).$$

Assume that if the game has multiple Nash equilibria, the equilibrium that produces the highest welfare $w(x)$ is chosen. In a Nash equilibrium of this game, the equilibrium investment $\hat{x}$ satisfies the first-order condition

$$\sum_{S \in \mathcal{P}(N)} \Pr(S \setminus \{i\} \mid \hat{x}_{-i})\ [r_i(S) - r_i(S \setminus \{i\})] = c_i'(\hat{x}_i) \tag{3}$$

for all $i \in N$. From this first-order condition, we see that any revenue schemes that feature the same value of the difference $r_i(S) - r_i(S \setminus \{i\})$ across all i and S will induce identical equilibrium research efforts. Therefore, for the rest of the paper, I will restrict attention to revenue schemes that pay the firms only if they succeed: $r_i(S) = 0$ if $i \notin S$. Comparing (3) with the first-best efforts (1) also shows that if the revenue $r_i(S)$ equals the incremental value $v(S) - v(S \setminus \{i\})$ for all i and S, then the system of equations characterized by (3) coincides with the welfare-maximizing first-order conditions. This correspondence reflects the notion that paying the incremental value aligns the social and private incentives.

However, it is not necessarily possible to pay the incremental values to all firms. In this paper, I consider a principal who sets a feasible revenue scheme to maximize the expected social surplus $w(x)$, given that the research efforts are the equilibrium (3) under the condition that external subsidies are not possible and the revenues that firms receive can only come from the value created by the technologies. A revenue scheme can then be thought of as a sharing rule for a joint project. In particular, I consider two types of such budget constraints. Under the *ex ante budget constraint*, the expected revenue given to firms must not exceed the expected value

$$\sum_{S \in \mathcal{P}(N)} \Pr(S \mid \hat{x}) \sum_{i \in S} r_i(S) \leq \sum_{S \in \mathcal{P}(N)} \Pr(S \mid \hat{x})\, v(S) \tag{4}$$

given the firms' equilibrium efforts $\hat{x}$ determined by the revenue scheme. Under the *ex post budget constraint*, the budget constraint exists in every possible state of the world:

$$\sum_{i \in N} r_i(S) \leq v(S) \tag{5}$$

for all $S \subseteq N$.

4 Analysis

4.1 Optimal revenue scheme under the ex ante budget constraint

To simplify the exposition, let $\bar{r}_i$ denote the expected revenue that firm i receives *if its research succeeds* given other firms' research efforts x_{-i},

$$\bar{r}_i = \sum_{S \in \mathcal{P}(N)} \Pr(S \setminus \{i\} \mid x_{-i})\; r_i(S), \tag{6}$$

and let $\bar{r}$ denote the vector $(\bar{r}_1, \ldots, \bar{r}_n)$. Given (2), firm i's expected profit can then be rewritten as $x_i \bar{r}_i - c(x_i)$ and the condition (3) for the equilibrium research efforts can be rewritten as

$$\bar{r}_i = c_i'(\hat{x}_i) \tag{7}$$

for all $i \in N$. The effort $\hat{x}_i$ that satisfies (7) is unique for each $\bar{r}_i$ and increasing in $\bar{r}_i$.

With the formulation, the principal's problem under the ex ante budget constraint becomes choosing $\bar{r}$ to maximize $w(\hat{x})$ subject to (7) and

$$\sum_{i \in N} \hat{x}_i \bar{r}_i \leq \bar{v}(\hat{x}).$$

This simplifies the problem to a standard contracting problem with one budget constraint.[8] Since (7) defines a one-to-one mapping between the expected revenue $\bar{r}_i$

[8] The structure in this model is similar to that of Holmström (1982), which can be characterized as

and the equilibrium effort $\hat{x}_i$, the problem can be further rewritten as a problem of choosing the equilibrium effort:

$$\max_{\hat{x}} \; w(\hat{x})$$
$$\text{subject to } \sum_{i \in N} \hat{x}_i c_i'(\hat{x}_i) \leq \bar{v}(\hat{x}). \tag{8}$$

Proposition 1 below describes a condition for the solution of this constrained maximization problem.

Proposition 1. *Under the ex ante budget constraint, the optimal expected revenue scheme $\bar{r}$ satisfies, for all $i \in N$,*

$$\frac{\partial \, \bar{v}(\hat{x}) / \partial \, \hat{x}_i - \bar{r}_i}{\bar{r}_i} = \frac{m}{e_i} \tag{9}$$

with some constant $m \geq 0$ and

$$e_i = \frac{c_i'(\hat{x}_i)}{\hat{x}_i c_i''(\hat{x}_i)}.$$

Proof. The Kuhn–Tucker necessary conditions imply that the solution to the maximization problem satisfies

$$\frac{\partial \bar{v}(\hat{x})}{\partial \hat{x}_i} - c_i'(\hat{x}_i) + \lambda \left[\frac{\partial \bar{v}(\hat{x})}{\partial \hat{x}_i} - c_i'(\hat{x}_i) - \hat{x}_i c_i''(\hat{x}_i) \right] = 0 \tag{10}$$

for all $i \in N$, and the complementary slackness condition

$$\lambda \left[\sum_{i \in N} \hat{x}_i c_i'(\hat{x}_i) - \bar{v}(\hat{x}) \right] = 0,$$

with $\lambda \geq 0$ being the Lagrange multiplier for the budget constraint. If the first-best efforts x^* are not feasible, then in the solution we have $\lambda > 0$ and $\partial \, \bar{v}(\hat{x}) / \partial \, \hat{x}_i - c'(\hat{x}_i) > 0$. Rearranging equation (10) yields

$$\frac{\partial \bar{v}(\hat{x}) / \partial \hat{x}_i - c'(\hat{x}_i)}{c'(\hat{x}_i)} = \frac{\lambda}{1 + \lambda} \cdot \frac{\hat{x}_i c''(\hat{x}_i)}{c'(\hat{x}_i)}.$$

follows. Given a vector of agents' actions x, a joint value of $w(x)$ is created. A principal specifies a sharing rule $s_i(w)$ such that $\sum_i s_i(w) \leq w$ for all w. Agent i's pay-off given the action profile x is $s_i(w(x)) - c(x_i)$. Comparing the constraints with $s_i(w(x))$ and $x_i \bar{r}_i$, the sharing rule $s_i(w(x))$ is not necessarily linear in x, while in this paper $x_i \bar{r}_i$ must be.

Letting $m = \lambda/(1+\lambda)$ and $c'(\hat{x}_i) = \bar{r}_i$ we arrive at equation (9).

If the first-best efforts are feasible, then $\partial\, \bar{v}(x^*)/\partial\, x_i - c_i'(x_i^*) = 0$ and equation (9) holds with $m = 0$. □

The result from this maximization problem is analogous to the Ramsey pricing formula, which in the simplest case is usually presented with a monopolist who supplies n independent goods (see Baumol and Bradford, 1970). The Ramsey pricing formula for good i, determining the deviation from the first-best marginal-cost pricing, is given by $(p_i - mc_i)/p_i = \mu/\varepsilon_i$, where p_i denotes the price of good i, mc_i its marginal cost, ε_i its price elasticity of demand, and μ a constant that is identical for all i. Equation (9) is the flip side of this formula with a monopsonist and n suppliers of research efforts. For all firms, the deviation of their marginal revenues $\bar{r}_i$ from their marginal contributions $\partial\, \bar{v}(\hat{x})/\partial\, \hat{x}_i$ is inversely proportional to the "elasticity" of effort that measures the change of equilibrium effort $\hat{x}_i$ in response to the change in the expected revenue $\bar{r}_i$.

4.2 Competitive benchmark

In this section, I define the competitive benchmark that captures the notion that royalties should reflect the hypothetical competition before the industry has committed to a standard. Then I present the conditions under which the research efforts induced by the benchmark are neither first-best nor second-best under the ex ante budget constraint.

Suppose that the revenue scheme is determined through the following competitive process after the innovation outcome is realized. The firms that have successfully invented their technologies engage in a Bertrand-like pricing game with a single user who values the technologies according to the function v. In state S, each firm $i \in S$ simultaneously proposes a price $r_i(S)$ to the user. The user then chooses a subset of technologies T to maximize the net value

$$\max_{T \in \mathcal{P}(S)} v(T) - \sum_{i \in T} r_i(S).$$

Each firm i's pay-off is the price $r_i(S)$ if its technology is in the set chosen by the buyer and zero otherwise.

A competitive benchmark for state S is defined as a price vector $r^c(S)$ that is part of a subgame-perfect equilibrium of the pricing game in which the buyer buys from all firms in the set S.[9] This definition of the competitive benchmark in this paper aligns with that of Lerner and Tirole (2015).[10]

The following Lemma 1 characterizes the competitive benchmark.

Lemma 1. *A price vector $r^c(S)$ is a competitive benchmark for state S if and only if*

$$\sum_{i\in T} r_i^c(S) \le v(S) - v(S \setminus T) \tag{11}$$

for all $T \subseteq S$, and for every firm $i \in S$, there exists a subset $T \subseteq S$ such that $i \in T$ and condition (11) *holds with equality.*

Proof. Given a price vector $r^c(S)$, buying the entire set S is optimal for the user if and only if (11) holds, since (11) is equivalent to

$$v(S \setminus T) - \sum_{i\in S\setminus T} r_i^c(S) \;\le\; v(S) - \sum_{i\in S} r_i^c(S)$$

for all $T \subseteq S$.

The next step is to show that the price vector is a competitive benchmark if and only if each firm faces at least one binding constraint in (11).

Suppose that in the equilibrium that constitutes a competitive benchmark, the condition (11) holds with equality for a subset T. If firm $i \in T$ increases its price by $\varepsilon > 0$, then for the subset T we have

$$\sum_{i\in T} r_i^c(S) + \varepsilon > v(S) - v(S \setminus T).$$

Rearranging this yields

$$v(S \setminus T) - \sum_{i\in S\setminus T} r_i^c(S) > v(S) - \sum_{i\in S} r_i^c(S) - \varepsilon.$$

[9] This condition rules out equilibria with coordination failure, in which firms with complementary technologies both choose too high prices. Alternatively, Lerner and Tirole (2015) impose that the competitive prices of technologies not bought by the user must be zero.

[10] In particular, the description given of the competitive prices in Lemma 1 is identical to equation (1) of Lerner and Tirole (2015) when there is no distinction between "within-" and "across-functionality" substitution and the demand is perfectly inelastic.

Since in this equilibrium, buying from the set S is optimal for the user, it follows that

$$v(S \setminus T) - \sum_{i \in S \setminus T} r_i^c(S) > v(T') - \sum_{i \in T'} r_i^c(S) - \varepsilon$$

for any subset T' such that $i \in T'$. This means if firm i unilaterally increases its price, the user will no longer buy technology i and it is not profitable for firm i to raise the price. Therefore, all firms face at least one such binding constraint, it is not possible for any firm to gain by unilaterally increasing its price.

For the other direction, if there exists $i \in S$ that faces no binding constraint, then there exists $\varepsilon > 0$ by which firm i can increase its price and not be dropped from the user's basket. □

Remark: The competitive benchmark as defined is not necessarily unique. Take for example the four-firm setting described in Section 2. In the state $\{A_1, B_1\}$ in which there are two perfect complements, any split of the full value to these two firms is a competitive benchmark, not just the symmetric one. We can impose further restrictions to narrow down the set of competitive benchmark, for example an assumption on symmetry or proportionality, but they are not necessary for the results that follow.

The inequality (11) represents the notion that the license fees that the firms can command for their technologies must be restricted by their incremental values. This incremental value rule applies not only to individual technologies but also to any subsets of technologies in that state. This means the competitive price for firm i may be strictly less than its *individual* incremental value $v(S) - v(S \setminus \{i\})$.

From (1) and (3), the efforts induced by the competitive benchmark are first-best if $\bar{r}_i^c$ equals the expected incremental value (1). Since Lemma 1 shows that $r_i^c(S)$ never exceeds the incremental value, the efforts induced by the competitive benchmark deviate from the first-best if $r_i^c(S) < v(S) - v(S \setminus \{i\})$ for some state S. Let $\hat{x}^c$ denote the equilibrium efforts given the competitive benchmark. Proposition 2 provides a sufficient condition for the competitive benchmark not to lead to first-best efforts.

Proposition 2. *If there exists a state S such that $\Pr(S \mid \hat{x}^c) > 0$ and for a subset $T \subseteq S$*

$$\sum_{i \in T} [v(S) - v(S \setminus \{i\})] > v(S) - v(S \setminus T), \tag{12}$$

then the first-best research efforts are not implemented by the competitive benchmark.

Proof. From (1) and (7), the first-best efforts x^* must satisfy

$$x_i^* \bar{r}_i = \sum_{S \in \mathcal{P}(N)} \Pr(S \mid x^*) \left[v(S) - v(S \setminus \{i\}) \right] \quad \text{for all } i \in N.$$

If there is a state S such that for $T \subseteq S$,

$$v(S) - v(S \setminus T) < \sum_{i \in T} \left[v(S) - v(S \setminus \{i\}) \right],$$

then by (11) we have

$$\sum_{i \in T} r_i^c(S) < \sum_{i \in T} \left[v(S) - v(S \setminus \{i\}) \right].$$

Define $\bar{r}^c$ analogously to (6). Multiplying both sides by $\Pr(S \mid \hat{x}^c)$ and sum over all states yields, with $\Pr(S \mid \hat{x}^c) > 0$,

$$\sum_{i \in T} \hat{x}_i^c \bar{r}_i^c < \sum_{S \in \mathcal{P}(N)} \Pr(S \mid \hat{x}^c) \sum_{i \in T} \left[v(S) - v(S \setminus \{i\}) \right].$$

This cannot be the case with the first-best efforts x^*, therefore $\hat{x}^c \neq x^*$ and the first-best efforts are not implemented by the competitive benchmark. □

The inequality (12) means that for the firms in the subset T, the sum of their individual contributions in state S exceeds their joint contribution. The restriction imposed by the joint incremental value means that at least some firms in the set T must get paid strictly less than their individual incremental value in the competitive benchmark for state S. Note that the result only requires this relationship in a single state.

Even in the case that the competitive benchmark does not achieve the first best, it may still implement the second-best efforts if the first-best efforts are not feasible. The following Proposition 3 provides the necessary condition that the competitive benchmark does not lead to the second-best efforts under the ex ante budget constraint. If the competitive benchmark is not on the boundary of the set of efforts that can be induced by a permissible revenue scheme, loosely speaking, there is “money” left on the table that can still be distributed, and increasing the revenues over the competitive benchmark can improve welfare.

Proposition 3. *If the first-best efforts are not implemented by the competitive benchmark and there exists a state* S *such that* $\Pr(S \mid \hat{x}^c) > 0$ *and for a partition* $\mathcal{T}$ *of* S

$$\sum_{T \in \mathcal{T}} [v(S) - v(S \setminus T)] < v(S), \tag{13}$$

then the second-best efforts under the ex ante budget constraint are not implemented by the competitive benchmark.

Proof. If the first-best efforts are feasible under the budget constraint, then they are also second-best and the proposition holds trivially.

Suppose that the first-best efforts are not feasible. From the assumption that there is a unique local and global maximum in $[0,1]^n$, if the first-best efforts are not in the feasible set, then there is no local maximum in the interior of the feasible set. Since a maximum must exist in the compact feasible set, the maximum is located on the boundary of the set. If the competitive benchmark corresponds to an interior point, it must not induce welfare-maximizing efforts in the feasible set, i.e. it does not implement the second-best research efforts. From the constraint (8), the efforts are interior if

$$\sum_{S \in \mathcal{P}(N)} \Pr(S \mid \hat{x}) \sum_{i \in N} r_i^c(S) < \sum_{S \in \mathcal{P}(N)} \Pr(S \mid \hat{x})\, v(S).$$

I will now show that the efforts induced by a competitive benchmark are interior under the condition described in this proposition. Since the sum of competitive prices in any state S does not exceed the value $v(S)$, the above strict inequality is true if and only if there exists a state S such that $\sum_i r_i^c(S) < v(S)$. If there exists a partition $\mathcal{T}$ of the set S such that (13) is true, then we have

$$\sum_{i \in S} r_i^c(S) \leq \sum_{T \in \mathcal{T}} [v(S) - v(S \setminus T)] < v(S)$$

where the first weak inequality follows from the condition of the competitive benchmark (11). □

With Proposition 2 and Proposition 3, we see that with complementary technologies, it is possible that there is room to improve welfare without breaking the budget. This happens if in some states the full value is not paid out in the competitive benchmark.

4.3 Ex post budget constraints

The previous sections consider the problem of providing innovation incentives under the single ex ante budget constraint (8) that is formulated in expectation across all states. Implementing a revenue scheme that only satisfies the ex ante constraint may require a principal who can absorb the risk. However, if such a mechanism does not exist and the principal is only allowed to distribute the available value within each state, then we need to consider the more stringent ex post constraints (5).

In Section 4.1 it is possible to simplify the problem to choosing the efforts x from the feasible set of efforts, since there is a one-to-one mapping between x_i and $\bar{r}_i$. However, the correspondence between a revenue scheme and the vector $\bar{r}$ is not unique, so multiple revenue schemes can be mapped to the same equilibrium efforts $\hat{x}$.

It should also be noted that even if the ex post budget constraints hold with equality for some states, it does not necessarily mean the budget is really "binding" in the sense it is not feasible to increase every firm's equilibrium effort. In the example in Section 2, Table 1 shows that the competitive prices use up the budget in all states except the state in which all firms are successful, but it is still possible to increase the expected revenue (and equilibrium efforts) for every firm by paying more than the competitive price of zero when all firms are successful.

The following result means that the problem with ex post budget constraints can be similarly converted to a similar problem with the set of feasible $\bar{r}$ (and thus x). The lemma restates the constraints (5) into a set of constraints on $\bar{r}$. This means the problem of choosing a revenue scheme that satisfies (5) can still be reduced to that of choosing the expected revenues $\bar{r}$ subject to the following set of constraints (14), without specifying the (not necessarily unique) implementation of the revenue scheme.

Lemma 2. *Given research efforts x and expected revenues $\bar{r}$, there exists a revenue scheme that implements $\bar{r}$ according to* (6) *and satisfies the constraints* (5) *for all states, if and only if $\bar{r}$ satisfies*

$$\sum_{i \in T} \hat{x}_i \bar{r}_i \leq \sum_{S \in \mathcal{Q}(T)} \Pr\left(S \mid \hat{x}\right) v(S), \tag{14}$$

for any subset $S \subseteq N$, where

$$\mathcal{Q}(T) = \{S \in \mathcal{P}(N) \mid S \cap T \neq \varnothing\}$$

is the set of states that has at least one of the firms in set S.

Proof. First, I will show that a revenue scheme that represents $\bar{r}$ and satisfies the ex post budget constraints (5) exists only if $\bar{r}$ and the corresponding $\hat{x}$ satisfy (14) for all S. From (6) and (2), a revenue scheme representing $\bar{r}$ implies

$$\sum_{i \in T} \hat{x}_i \bar{r}_i = \sum_{S \in \mathcal{P}(N)} \Pr(S \mid \hat{x}) \sum_{i \in T} r_i(S)$$

for all $T \subseteq N$. The imposition $r_i(S) = 0$ if $i \notin S$ implies that

$$\sum_{i \in T} r_i(S) \leq \sum_{i \in S} r_i(S)$$

for any S and T, and it also means we can safely ignore the states that do not intersect with T, yielding

$$\sum_{i \in T} \hat{x}_i \bar{r}_i \leq \sum_{S \in \mathcal{Q}(T)} \Pr(S \mid \hat{x}) \sum_{i \in S} r_i(S)$$

for all $T \subseteq N$. If the revenue scheme satisfies the ex post budget constraint (5), then

$$\sum_{i \in T} \hat{x}_i \bar{r}_i \leq \sum_{S \in \mathcal{Q}(T)} \Pr(S \mid \hat{x})\, v(S)$$

for all $T \subseteq N$. Therefore, a scheme represents $\bar{r}$ and satisfies (5) only if $\bar{r}$ and $\hat{x}$ satisfy (14).

Next, I will show that there exists a revenue scheme that satisfies the ex post budget constraints if $\bar{r}$ and $\hat{x}$ satisfy (14). The existence of a revenue scheme that represents $\bar{r}$ and satisfies the ex post constraint (5) for all states means the minimum in the following constrained minimization problem is zero:

$$\min_{r} \sum_{i \in N} \left[\hat{x}_i \bar{r}_i - \sum_{S \in \mathcal{P}(N)} \Pr(S \mid \hat{x})\, r_i(S) \right]^2$$
$$\text{subject to (5) for all } S \subseteq N$$
$$\text{and } r_i(S) \geq 0 \text{ for all } i \in N \text{ and } S \subseteq N.$$

I will show by contrapositive that if the constrained minimum is not zero, then condition (14) is violated.

Suppose the constrained minimum is strictly greater than zero and is attained at

r^0. The Kuhn–Tucker necessary conditions for this minimization problem are

$$2\Pr(T \mid \hat{x})\left[\hat{x}_i\bar{r}_i - \sum_{S\in\mathcal{P}(N)}\Pr(S \mid \hat{x})\, r_i(S)\right] + \lambda_{iT} = \mu_T \quad \text{for all } i \in N \text{ and } T \subseteq N \tag{15}$$

$$\mu_T\left[\sum_{i\in T} r_i(T) - v(T)\right] = 0 \qquad \text{for all } T \subseteq N \tag{16}$$

$$\lambda_{iT}\, r_i(T) = 0 \qquad \text{for all } i \in N \text{ and } T \subseteq N \tag{17}$$

with the multipliers $\mu_T \geq 0$ for each ex post budget constraint and $\lambda_{iT} \geq 0$ for each non-negativity constraint.

Since $\hat{x}_i\bar{r}_i \geq 0$ and we can always decrease $r_i(S)$ down to zero, we can focus on the case that

$$\hat{x}_i\bar{r}_i > \sum_{S\in\mathcal{P}(N)}\Pr(S \mid \hat{x})\, r_i^0(S)$$

for some $i \in N$. Let A be the set of every such i,

$$A = \left\{ i \in N \mid \hat{x}_i\bar{r}_i > \sum_{S\in\mathcal{P}(N)}\Pr(S \mid \hat{x})\, r_i^0(S)\right\}.$$

Equation (15) implies $\mu_T > 0$ for every T such that $T \cap A \neq \varnothing$. By (16), the ex post budget constraints must be binding for all such sets T. Given $\mu_T > 0$, we have that $\lambda_{jT} > 0$ for $j \notin A$ since the first term in (15) is zero, which then implies through (17) that $r_j(T) = 0$. This means, for $j \in A$ and T such that $T \cap A \neq \varnothing$,

$$\sum_{j\in A} r_j^0(T) = v(T)$$

and

$$\sum_{S\in\mathcal{P}(N)}\Pr(S \mid \hat{x})\sum_{j\in A} r_j^0(S) = \sum_{T\in\mathcal{Q}(A)}\Pr(T \mid \hat{x})\, v(T).$$

From the definition of set A, we then have

$$\sum_{j\in A}\hat{x}_j\bar{r}_j > \sum_{T\in\mathcal{Q}(A)}\Pr(T \mid \hat{x})\, v(T)$$

which contradicts condition (14). Therefore, if $\bar{r}$ satisfies (14), the minimum must be zero, which means there exists a feasible revenue scheme that represents $\bar{r}$. $\square$

Applying $S = N$ to inequality (14) yields the relaxed budget constraint (8), so the constraints on other subsets of N restrict the share of $\bar{v}(\hat{x})$ that can be given to firms in those subsets. The intuition of Lemma 2 is that the maximum expected payment a principal can assign to a group of firms is the portion of the available expected value $\bar{v}(\hat{x})$ that comes from states that have at least some firms in that group. If all the value in any states with at least one firm from set S is used, then it is not possible to pay more to firms in S without taking it from another firm in the set.[11] When the values of $\hat{x}$ and $\bar{r}$ satisfy the whole stack of such conditions, it is possible to find a revenue scheme that requires no transfers across different states, as prescribed by (5).

With constraints defined by expected revenues, the problem is now in a form similar to the problem considered in previous sections. A principal chooses expected revenues $\bar{r}$, subject to the constraints (14), to maximize the surplus $w(\hat{x})$ given the investment equilibrium (7). It is still the case that if the efforts are not first-best and are interior in the feasible set, then the efforts are also not the constrained optimal or second-best efforts.

Proposition 3 still directly applies in this case, with the constraint (8) replaced by the set of constraints (14); the expected surplus from the competitive benchmark is not optimal, neither in the first-best nor second-best senses, if some technologies are complementary and the benchmark research efforts are interior in the feasible set defined by (14). The proof of Proposition 3 applies to a compact feasible set, it holds for the feasible set defined by these budget constraints.

However, given that the feasible set defined by (14) is more restricted than (8), it remains to be seen whether the efforts that are interior under the laxer constraint are also interior in the set defined by (14). The following Proposition 4 shows that a competitive benchmark that is interior in the feasible set defined by (8) is also interior in the feasible set defined by (14). Thus, Proposition 3 still holds.

Proposition 4. *Given $\hat{x}$ such that $\hat{x}_i > 0$ for all i, if the expected revenue from a competitive benchmark is interior in the feasible region defined by the constraint* (8)*, it is also interior in the feasible region defined by the constraints* (14)*.*

Proof. Consider the research efforts induced by competitive benchmark $r_i^c(S)$ that

[11] Note that the formulation of $\bar{r}_i$ works only under the assumption that firm i does not receive any reward when it fails. Paying firm i when it fails does not increase its incentive to invest as shown by (3).

are interior in the feasible region defined by (8). The following steps show that the efforts induced by a competitive benchmark cannot be interior in the feasible region of (8) but not interior in the feasible region of (14).

From the definition of expected revenue, the competitive benchmark being interior under constraint (8) means

$$\sum_{T\in\mathcal{P}(N)} \Pr\left(T \mid \hat{x}^c\right) \sum_{i\in N} r_i^c(T) < \sum_{T\in\mathcal{P}(N)} \Pr\left(T \mid \hat{x}^c\right)\, v(T), \tag{18}$$

while not being interior in (14) means, given that the competitive benchmark satisfies (11), the inequality (14) holds with equality for some state $S \in \mathcal{P}(N)$ such that $S \neq N$, that is

$$\sum_{T\in\mathcal{Q}(S)} \Pr\left(T \mid \hat{x}\right) \sum_{i\in S} r_i^c(T) = \sum_{T\in\mathcal{Q}(S)} \Pr\left(T \mid \hat{x}\right) v(T). \tag{19}$$

From Lemma 1, equation (19) is true only if

$$\sum_{i\in S\cap T} r_i^c(T) = v(T) \tag{20}$$

for all $T \in \mathcal{Q}(S)$. That is, only firms in set S can receive a positive revenue in any states in the set $\mathcal{Q}(S)$ and the values in those states are fully distributed. Subtracting (19) from (18) yields

$$\sum_{T\in\mathcal{P}(N)\setminus\mathcal{Q}(S)} \Pr\left(T \mid \hat{x}\right) \sum_{i\in N\setminus S} r_i^c(T) < \sum_{T\in\mathcal{P}(N)\setminus\mathcal{Q}(S)} \Pr\left(T \mid \hat{x}\right) v(T).$$

This is true only if there exists a set T such that $T \cap S = \varnothing$ and

$$0 \leq \sum_{i\in T} r_i^c(T) < v(T).$$

Consider a set U such that $T \subseteq U$ and $U \cap S \neq \varnothing$. From Lemma 1, the competitive benchmark satisfies, given that $v(T) > 0$,

$$\sum_{i\in U\setminus T} r_i^c(U) \leq v(U) - v(T) < v(U).$$

But since $U \in \mathcal{Q}(S)$, then by (20) we should have

$$\sum_{i \in S \cap U} r_i^c(U) = v(U)$$

with $S \cap U$ being disjoint from T. Therefore, it is not possible that the expected revenue from a competitive benchmark is interior under (8), but not under (14). □

The idea of the proof is that for a constraint (14) to be binding for a set of firms S, the values in all states that contain at least one firm in S must be fully paid only to the firms in set S. However, some of the states also include firms outside of set S that have positive values by themselves. Since the revenues in the competitive benchmark are limited by (joint) incremental values, it cannot be the case that the full values in those states are distributed to only the firms in set S.

Proposition 4 means that while the extra state-based budget constraints considered in this section potentially reduce the feasible set, they do not affect the main results on the competitive benchmark described in Proposition 2 and Proposition 3. It is possible that, with complementary technologies, the competitive benchmark does not lead to the welfare-maximizing research efforts under the condition that the incentives given to the firms can only be from the values created by the available technologies in a particular state.

Numerical example Consider again the four-firm model described in Section 2, with the value function

$$v(S) = \begin{cases} 1 & \text{if } S \cap \{A_1, A_2\} \neq \varnothing \text{ and } S \cap \{B_1, B_2\} \neq \varnothing \\ 0 & \text{otherwise} \end{cases}$$

and let the cost function be $c(x_i) = 0.2x_i^2$. In the second-best case, the budget constraint $\sum_{i \in N} \hat{x}_i \bar{r}_i \leq \bar{v}(\hat{x})$ is binding, with the solution being $\hat{x} = 0.312$. This gives $\bar{r}_i = 0.222$, $\bar{v}(\hat{x}) = 0.277$, and $w(\hat{x}) = 0.164$. The symmetric competitive benchmark yields the expected welfare of 0.159, which is 3.25 per cent lower than the second-best welfare.

5 Discussion

In this paper, I analyze incentives for innovation in the context of complex technologies with multiple interacting inventions. The model allows us to study how the value jointly created by the inventions should optimally be appropriated by their inventors and how the competitive outcome, as commonly defined, performs compared to the optimal rule.

The paper shows that allowing supra-competitive rewards for innovators may enhance economic welfare. Specifically, it increases the expected welfare by allowing innovators to reap more benefits when there are competing substitutes that drive the rewards down in order to compensate for the suboptimal incentives that arise from complementarities. This result provides a caveat to the attractive notion in the standard-related intellectual property rights policy that SEP holders should be restricted to charging pre-standardization competitive prices (Swanson and Baumol, 2005). The presence of complementarities in complex innovations is well-recognized in the literature (e.g. Shapiro, 2001; Ménière, 2008; Denicolò and Halmenschlager, 2012; Spulber, 2016; Denicolò and Zanchettin, 2022). For example, a standard for mobile telephony requires inventions related to, among other things, radio communications, e.g. modulation, and resource management, e.g. power control (Goodman and Myers, 2005), which are likely complementary in nature.

While the results in this paper show that the competitive outcome is not optimal in plausible settings of complex technologies, it is not necessarily apparent how the second-best reward scheme should be implemented as formulating the optimal reward scheme requires more information than the competitive outcome. The incremental value rule rewards firms solely based on the characteristics of the invented technologies, i.e. the rewards when the set of invented technologies is S can be computed based on observing $v(T)$ for all $T \subseteq S$ and nothing else. The authority may be able to observe that the incremental value rule is not optimal based on the invented technologies. For example, under the setting described in Section 2, the authority can observe the presence of both complements and substitutes in the state in which all technologies are invented. However, how much the optimal rewards deviate from the incremental values depends on the innovation cost functions. If the first-best solution is interior in the feasible set, then the optimal reward scheme does not distribute the entire value to the innovators. If instead the first best is not feasible, then the second-best reward scheme distributes the entire value to the innovators. Whether the first-best solution is feasible depends on the cost function. The second reward scheme may also depend on the unobserved failed technologies. While it may be impractical for the social planner to fully acquire all the required information to formulate the optimal reward scheme, the planner may at least exercise caution in adhering to the competitive benchmark rule, especially in the case that complementarity is already observed among the invented technologies.

Under this paper's framework, we can also analyze other proposed rules for dividing the values among the innovators. One such rule is that each patent earns its inventor an equal share of the total value, a rule implemented by several patent pools (Layne-Farrar and Lerner, 2011). In the context of this paper's model, dividing $v(S)$ equally among the successful firms in state S is generally not optimal. Alternatively, Layne-Farrar et al. (2007) propose that the value of a standard is divided according to the Shapley value (Shapley, 1953).

In general, rewarding the innovators according to the Shapley value in each realized state of the world does not coincide with the second best. Firstly, the Shapley value by design distributes the entire value in each state, which, as previously mentioned, is not necessarily optimal. Even when the second best prescribes distributing the entire value to the innovators, the Shapley value does not necessarily induce the same investment as the second best. However, at least under some settings, the Shapley value is a numerically good approximation of the second-best scheme, even when the competitive benchmark also distributes the entire value.[12] Given that the Shapley value requires only the knowledge of the value function, similar to the competitive benchmark, it would be an interesting question for future research to find the conditions under which the Shapley value is a good approximation of the second-best scheme.

The effects illustrated in this paper should be interpreted as one of many relevant factors that affect innovation incentives. Since the model assumes that the technologies generate fixed welfare that is unaffected by royalties, it shuts down effects associated with non-perfectly-inelastic demand such as the deadweight loss from royalties and the multiple marginalization problem (also known as royalty stacking

[12] For example, suppose that a standard requires two components, as in Section 2, but there is only one potential inventor for the second component B so that $v(S) = 1$ if $S \cap \{A_1, A_2\} \neq \varnothing$ and $B_1 \in S$ and $v(S) = 0$ otherwise. With the cost function $c(x_i) = 0.2x_i^2$, the expected welfare under the Shapley value rule is approximately 0.001 percent less than the second-best welfare, whereas the expected welfare under the symmetric competitive benchmark is approximately 1.2 percent less than the second-best welfare.

in the patent licensing literature) whereby the cumulative royalties demanded by multiple patent holders exceed even the monopoly level.

The model assumes that each firm simultaneously pursue a distinct, independently patentable invention, which follows from how the notion of incremental value is discussed in the literature (see e.g. Farrell et al., 2007; Layne-Farrar et al., 2014). In the case of technical standards, this modeling choice may be a reasonable approximation of the situation where there are multiple means to achieve a certain functionality and each firm pursues a different mean (see e.g. Farrell et al., 1992), and the approach that each firm takes may be influenced by factors such as existing expertise that the firms possess. There are ways that the assumptions can be relaxed. If multiple firms pursue the same technology with only one winner obtaining the patent (for example, there is a common idea of how a functionality might be achieved), a situation known as a patent race, firms may have the incentives to over-invest in research.[13]

Multiple technologies can also be invented by a single firm. In the model, if multiple prospective technologies belong to a single firm, then the firm can internalize the externalities between the technologies. This can be welfare-improving in the case of complementary technologies. However, if the technologies belonging to the same firm are substitutes, then the firm may have the incentives to under-invest if the firm is rewarded based on the competitive benchmark at the individual technology level.[14] This modeling approach can also be used to analyze the case where each firm can choose between multiple research directions, with the different directions corresponding to imperfect substitute technologies. Multiple research directions may also include, for example, the choice between pursuing a substitute or a complement to another firm's technology. If a firm can independently pursue both directions at the same time, then the model would suggest that the investment in the complementary technology may be too low compared to the actual contribution to society. However, if the firm has to decide on a single direction, it might be the case that allowing supra-competitive rewards for substitute technologies may make investing in the substitute technology relatively more attractive, diverting the investment away from the valuable complementary technology.

[13] See Tandon (1983) and Wright (1983) for probability-of-discovery models of patent race.

[14] The alignment between the private and social values can be restored by paying the firm the incremental contribution at the firm level, but this involves allowing more value to be appropriated by the firm.

References

Baron, J. and Pohlmann, T. (2018). Mapping standards to patents using declarations of standard-essential patents. *Journal of Economics & Management Strategy*, 27(3):504–534.

Baumol, W. J. and Bradford, D. F. (1970). Optimal departures from marginal cost pricing. *American Economic Review*, 60(3):265–283.

Biagi, A. and Denicolò, V. (2014). Timing of discovery and the division of profit with complementary innovations. *Journal of Economics & Management Strategy*, 23(1):89–102.

Che, Y.-K. and Gale, I. (2003). Optimal design of research contests. *American Economic Review*, 93(3):646–671.

Choi, J. P. and Gerlach, H. (2015). Patent pools, litigation, and innovation. *RAND Journal of Economics*, 46(3):499–523.

D'Antoni, M. and Rossi, M. A. (2014). Appropriability and incentives with complementary innovations. *Journal of Economics & Management Strategy*, 23(1):103–124.

Dehez, P. and Poukens, S. (2014). The Shapley value as a guide to FRAND licensing agreements. *Review of Law & Economics*, 10(3):265–284.

Denicolò, V. and Halmenschlager, C. (2012). Optimal patentability requirements with complementary innovations. *European Economic Review*, 56(2):190–204.

Denicolò, V. and Zanchettin, P. (2022). Patent protection for complex technologies. *International Journal of Industrial Organization*, 81:102811.

Dewatripont, M. and Legros, P. (2013). 'Essential' patents, FRAND royalties and technological standards. *Journal of Industrial Economics*, 61(4):913–937.

Farrell, J., Hayes, J., Shapiro, C., and Sullivan, T. (2007). Standard setting, patents, and hold-up. *Antitrust Law Journal*, 74(3):603–670.

Farrell, J., Shapiro, C., Nelson, R. R., and Noll, R. G. (1992). Standard setting in high-definition television. *Brookings Papers on Economic Activity. Microeconomics*, 1992:1–93.

Federal Trade Commission (2011). The evolving IP marketplace: Aligning patent notice and remedies with competition. Technical report.

Ganglmair, B., Froeb, L. M., and Werden, G. J. (2012). Patent hold-up and antitrust: How a well-intentioned rule could retard innovation. *Journal of Industrial Economics*, 60(2):249–273.

Gilbert, R. J. and Katz, M. L. (2011). Efficient division of profits from complementary innovations. *International Journal of Industrial Organization*, 29(4):443–454.

Goodman, D. J. and Myers, R. A. (2005). 3G cellular standards and patents. In *2005 International Conference on Wireless Networks, Communications and Mobile Computing*, volume 1, pages 415–420, Maui, HI.

Holmström, B. (1982). Moral hazard in teams. *Bell Journal of Economics*, 13(2):324–340.

Hougaard, J. L., Ko, C. Y., and Zhang, X. (2023). A conceptual model for FRAND royalty setting. *Mathematical Social Sciences*, 123:167–176.

Layne-Farrar, A. and Lerner, J. (2011). To join or not to join: Examining patent pool participation and rent sharing rules. *International Journal of Industrial Organization*, 29(2):294–303.

Layne-Farrar, A. and Llobet, G. (2014). Moving beyond simple examples: Assessing the incremental value rule within standards. *International Journal of Industrial Organization*, 36:57–69.

Layne-Farrar, A., Llobet, G., and Padilla, J. (2014). Payments and participation: The incentives to join cooperative standard setting efforts. *Journal of Economics & Management Strategy*, 23(1):24–49.

Layne-Farrar, A., Padilla, A. J., and Schmalensee, R. (2007). Pricing patents for licensing in standard-setting organizations: Making sense of FRAND commitments. *Antitrust Law Journal*, 74(3):671–706.

Lemley, M. A. (2002). Intellectual property rights and standard-setting organizations. *California Law Review*, 90(6):1889–1980.

Lerner, J. and Tirole, J. (2015). Standard-essential patents. *Journal of Political Economy*, 123(3):547–586.

Ménière, Y. (2008). Patent law and complementary innovations. *European Economic Review*, 52(7):1125–1139.

Pentheroudakis, C. and Baron, J. (2017). Licensing terms of standard essential patents: A comprehensive analysis of cases. JRC Science for Policy Report EUR 28302 EN.

Pigou, A. C. (1920). *The Economics of Welfare*. Macmillan, London.

Shapiro, C. (2001). Navigating the patent thicket: Cross licenses, patent pools, and standard setting. In Jaffe, A. B., Lerner, J., and Stern, S., editors, *Innovation Policy and the Economy*, volume 1, pages 119–150. MIT Press, Cambridge, MA.

Shapiro, C. (2007). Patent reform: Aligning reward and contribution. In Jaffe, A. B., Lerner, J., and Stern, S., editors, *Innovation Policy and the Economy*, volume 8, pages 111–156. University of Chicago Press, Chicago, IL.

Shapley, L. S. (1953). A value for n-person games. In *Contributions to the Theory of Games*, volume 2, pages 307–318. Princeton University Press, Princeton, NJ.

Siebrasse, N. V. and Cotter, T. F. (2017). Judicially determined FRAND royalties. In Contreras, J. L., editor, *The Cambridge Handbook of Technical Standardization Law: Competition, Antitrust, and Patents*, pages 365–388. Cambridge University Press, Cambridge.

Spulber, D. F. (2016). Patent licensing and bargaining with innovative complements and substitutes. *Research in Economics*, 70(4):693–713.

Swanson, D. G. and Baumol, W. J. (2005). Reasonable and nondiscriminatory (RAND) royalties, standards selection, and control of market power. *Antitrust Law Journal*, 73(1):1–58.

Tandon, P. (1983). Rivalry and the excessive allocation of resources to research. *Bell Journal of Economics*, 14(1):152–165.

Taylor, C. R. (1995). Digging for golden carrots: An analysis of research tournaments. *American Economic Review*, 85(4):872–890.

Wright, B. D. (1983). The economics of invention incentives: Patents, prizes, and research contracts. *American Economic Review*, 73(4):691–707.